s Domain Analysis

Moments

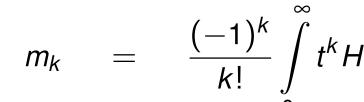
In case of input being $\delta(t)$, the r transfer function itself; since Lap function is UNITY

Characteristic of Impulse Response

$$H(s) = \int_{0}^{\infty} H(t) e^{-st} dt$$
(3)
= $\int_{0}^{\infty} H(t) \left(1 - st + s^{2} \frac{t^{2}}{2} + \dots + s^{k} \frac{(-1)^{k} t^{k}}{k!} + \dots \right) dt$ (4)
= $\int_{0}^{\infty} \sum_{k=0}^{\infty} s^{k} \frac{(-1)^{k} t^{k}}{k!} H(t) dt$ (5)
= $\sum_{k=0}^{\infty} s^{k} \frac{(-1)^{k}}{k!} \int_{0}^{\infty} t^{k} H(t) dt$ (6)



Comparing 1 and 6,



Pade Approximation

$$H_{p,q} = \frac{P(s)}{O(s)} = \frac{a_0 + a_1 + \dots + a_p s^p}{1 + b_1 + \dots + b_p s^q}$$
(10)

$$H(s) = -\sum_{n=1}^{\infty} m_{n} s^{k} \text{ limited to first } (n+q+1) \text{ terms}$$
(11)

$$H(s) = \sum_{k=0}^{\infty} m_k s^{n} \text{ limited to first (p+q+1) terms}$$
(11)

$$= m_0 + m_1 s + m_2 s^2 + \dots + m_{p+q} s^{p+q} + r(s) s^{p+q+1}$$
(12)

$$= H_{p,q}(s) + O(s^{p+q+1})$$
(13)

$$a_0 + a_1 + \dots + a_p s^p = (1 + b_1 + \dots + b_q s^q) \\ \times (m_0 + m_1 s + m_2 s^2 + \dots + m_{p+q} s^{p+q} + r(s) s^{p+q+1})$$

Deriving Moments from MNA Formulation

$$\begin{aligned} \mathbf{M}\dot{\mathbf{X}}(t) &= -G\mathbf{X}(t) + P\mathbf{U}(t) \end{aligned} \tag{14} \\ \mathbf{Y}(t) &= Q\mathbf{X}(t) \end{aligned} \tag{15}$$

Assuming that $\mathbf{X}(0) = 0$, taking Laplace transform of above,

$$sMX(s) = -GX(s) + PU(s)$$

 $Y(s) = QX(s)$

$$X(s) = (G+sM)^{-1} P U(s)$$

$$Y(s) = QX(s)$$

$$= Q(G+sM)^{-1} P U(s)$$

$$\Rightarrow H(s) = Q(G+sM)^{-1} P$$

Deriving Moments from MNA Formulation ...

$$\Rightarrow$$
 H(s) = Q(G+sM)^{-1}P

Here, coefficients of Maclaurin expansion of H(s) are given by,

$$M_j = (-1)^j Q (G^{-1}M)^j G^{-1}P$$

Computation of moments requires G to be invertible. This requirement is easily satisfied by most interconnect circuits in which each node has a DC path to the ground.

Deriving Moments from for RLC circuits

Lets consider all entries in unknown vector $\mathbf{X}(s)$ as outputs, i.e. Q is an identity matrix,

$$\mathbf{M}_{0} = G^{-1}PU \mathbf{M}_{1} = G^{-1}MG^{-1}P = G^{-1}M\mathbf{M}_{0} \mathbf{M}_{2} = (G^{-1}M)^{2}G^{-1}M = G^{-1}M\mathbf{M}_{1} \dots$$

$$GM_0 = PU$$

$$GM_1 = MM_0$$

$$GM_2 = MM_1$$

$$\dots$$

$$GM_{i+1} = MM_i$$

Moments are to be evaluated iteratively,

Matrix *G*, is admittance matrix of resistive tree derived from original RLC tree, by removing all *C* and *L*. Solving $G\mathbf{M}_0 = PU$ implies obtaining DC solution, which is very straight-forward.

Next, supposing \mathbf{M}_i is given, lets compute \mathbf{M}_{i+1}

 $G\mathbf{M}_{i+1} = M\mathbf{M}_i$

 \Rightarrow tree remains same (as *G* if on LHS)

Inputs are changed to $M\mathbf{M}_i$, M is interpretable BUT \mathbf{M}_i needs further interpretation

Deriving Moments from for RLC circuits ...

 $M \equiv \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix}$, as *C* imples KCL (currents through capacitors) & *L* KVL (voltages across inductors) We partition *M***M**_{*i*} according to the composition of *M*

$$M\mathbf{M}_{i} = \begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} M_{iV} \\ M_{il} \end{bmatrix} \equiv \begin{bmatrix} I_{C} \\ V_{L} \end{bmatrix}$$

Entries in I_C are related to product of capacitance and the i^{th} moment of voltage (M_{iV})

Entries in V_L are realted to product of inductance and the i^{th} moment of voltage (M_{il})

We generate NEW tree from OLD tree

zeroing out oldl sources adding CURRENT sources and VOLTAGE sources at location of capacitors and inductors of original tree

The solution is now trivial